Exam Seat No:________________ Enrollment No:____________________

C.U.SHAH UNIVERSITY

Wadhwan City

Subject Code : 5SC02MTC2 **Summer Examination-2014** **Date: 11/06/2014**

Subject Name : Partial Differential Equations

Branch/Semester:- M.Sc(Mathematics) /II **Time:2:00To 5:00**

Examination: Regular

Instructions:-

(1) Attempt all Questions of both sections in same answer book / Supplementary

(2) Use of Programmable calculator & any other electronic instrument is prohibited.

(3) Instructions written on main answer Book are strictly to be obeyed.

(4) Draw neat diagrams & figures (If necessary) at right places

(5) Assume suitable & Perfect data if needed

SECTION-I

Q-1 a) Find the particular integral of $(3D - D' - 1)z = e^{x-y}$ (02)

b) Find a partial differential equation by eliminating a and b from the equation $z = ax + by + a^2 + b^2$. (02)

c) Find solution of $\frac{\partial^3 z}{\partial x^3} = 0$. (01)

d) Find order and degree of the equation $\frac{\partial^2 z}{\partial x^2} + 2xy\left(\frac{\partial z}{\partial x}\right)^2 + \frac{\partial z}{\partial y} = 5$. (01)

e) $4r - s + yt - xyp + q = xy^3$is a linear partial differential equation. Determine whether this statement is true or false. (01)

Q-2 a) Solve: $(D + D' + 1)z = e^{-x}$. (05)

b) Eliminate the arbitrary functions and obtained a partial differential equation$z = f(x^2 - y) + g(x^2 + y)$. (05)

c) Prove that $F(D, D')e^{ax+by} = F(a, b)e^{ax+by}$, where a and b are constants. (04)

OR

Q-2 a) Obtain the solution of the following equation (05)

i) $(5D - 3D' + 2)z = 0$,

ii) $(D^2 - DD' - 2D'^2)z = 0$.

b) Prove that $F(D, D')[e^{ax+by}g(x, y)] = e^{ax+by}[F(D + a, D'b]g(x, y)$, where a and b are constants. (05)

c) Find the particular integral of $(D^2 - D'^2)z = x - y^2$. (04)

Q-3 a) Reduce the equation $4r = t$ to Canonical form and hence solve it. (07)

b) Solve: $(x^2D^2 - 3xyDD' + 2y^2D'^2 + xD + 2yD')z = x + 2y$. (07)

OR

Q-3 a) Reduce the equation $y^2r - 2xys + x^2t = \frac{y^2}{x}p + \frac{x^2}{y}q$ to Canonical form. (07)

b) Solve: $(2D - D' + 3)z = \cos(3x - y)$. (07)

SECTION-II

Q-4 a) Write the Laplace equation in spherical co-ordinate system. (02)

b) $xu_x + yu_y = u^2$ is a nonlinear partial differential equation. Determine whether the statement is true or false. (01)

c) Laplace equation is considered in the Dirichlet BVP. Determine whether the statement is true or false. (01)

d) Write one-dimensional wave equation. (01)

e) The Poisson Integral formula can be obtained from Neumann BVP. Determine whether the statement is true or false. (01)

f) Using which method one can solve nonlinear partial differential equation. (01)

Q-5 a) Solve the equation $u_{xx} = \frac{1}{c^2} u_{tt}$ $(-\infty < x < \infty)$ subject to the conditions, $u(x,t) \& \left(\frac{\partial u}{\partial x}\right)_{(x,t)} \to 0$ as $x \to \pm\infty$, and $u(x,0) = f(x)$, $\left(\frac{\partial u}{\partial t}\right)_{(x,0)} = 0$. (07)

b) In usual notation, prove that $\nabla^2 \psi = \frac{\partial^2 \psi}{\partial r^2} + \frac{1}{r} \cdot \frac{\partial \psi}{\partial r} + \frac{1}{r^2} \cdot \frac{\partial^2 \psi}{\partial \theta^2} + \frac{\partial^2 \psi}{\partial z^2}$. (07)

OR

Q-5 a) Solve interior Dirichlet problem for a function $\phi = \phi(r, \theta)$ for a circle and show that solution is of the form (07)
$\phi(r,\theta) = \sum_{n=0}^{\infty} r^n (A_n \cos n\theta + b_n \sin n\theta)$,with A_n, B_n constants.

b) State and prove Harnack's theorem. (07)

Q-6 a) Using Monge's method, solve the equation $r + 4s + 3t = xy$. (07)

b) Solve $\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 \psi}{\partial t^2} = 0$, by method of separation variable and show that solution is $\psi(x,y,z,t) = e^{\pm i(lx+my+nz+kct)}$, where l, m, n, k are constants with $l^2 + m^2 + n^2 = k^2$. (07)

OR

Q-6 a) Using Monge's method, solve the equation $rt - s^2 + 1 = 0$. (07)

b) Solve $\frac{\partial^2 \psi}{\partial r^2} + \frac{1}{r} \cdot \frac{\partial \psi}{\partial r} + \frac{1}{r^2} \cdot \frac{\partial^2 \psi}{\partial \theta^2} + \frac{\partial^2 \psi}{\partial z^2} - \frac{1}{k} \cdot \frac{\partial \psi}{\partial t} = 0$, by method of separation variable and show that solution is (07)
$(x,y,z,t) = J_n(mr) e^{\pm in\theta \pm lz - k\lambda^2 t}$.

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